# 3D Models for Calculus

During the Fall semester of 2025, our faculty member Dr. Cary Lee took a sabbatical leave to explore the possibility of using 3D printers to make models for calculus education.

The result is very satisfactory; he managed to master the sophisticated software “Blender” to create computer files for complex mathematical objects (not just objects usually seen in mechanical engineering); and then use 3D printers to make precise renditions. Each model is then hand-sanded, patched with epoxy (if necessary) to produce a high quality finish.

The following is a catalogue of the models he made. Each model is purposely colored, complete with the *xy*-plane, labelled axes, explanations, and even calculations when appropriate. Each model is dissectible in the sense that some pieces can be removed to show hidden detail or to provide different perspectives.

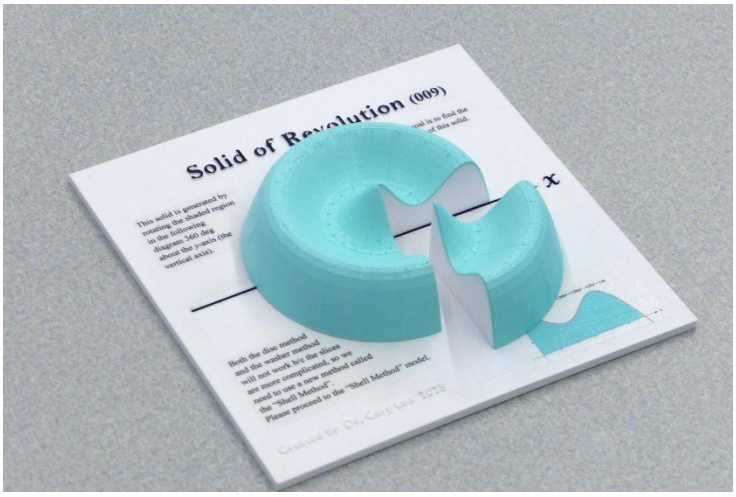
The models displayed here are just a few examples taken from different topics to test the capabilities of 3D printing, as well as the effectiveness of the models in teaching. They are by no means representing every topic, nor the most representational in any specific area of calculus.

If we find that they are beneficial to our students, more models will be built.

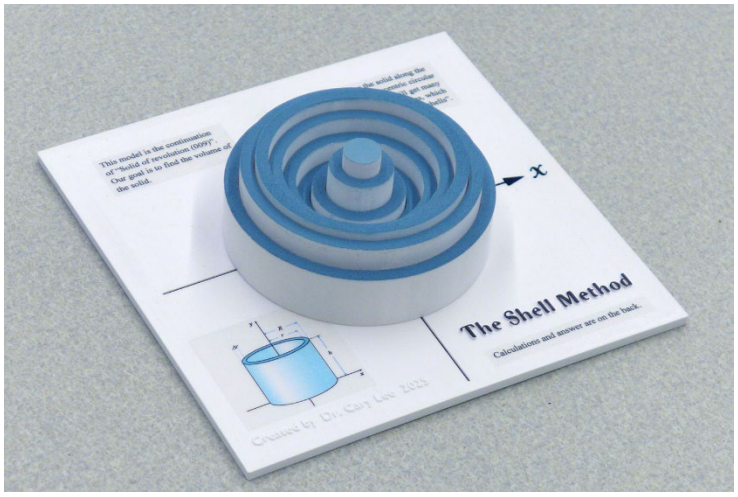
Please contact the department chairs for any questions or suggestions.

**Part I  Models for Solids of Revolution.**

a) Solid of revolution and the shell method

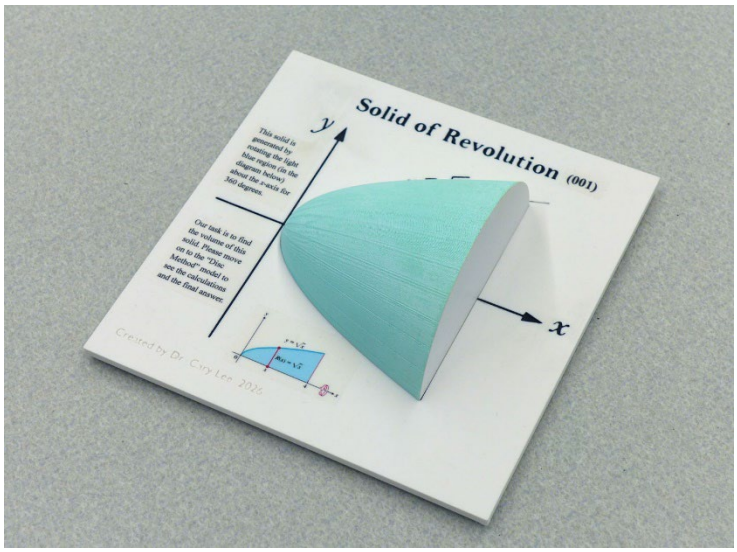


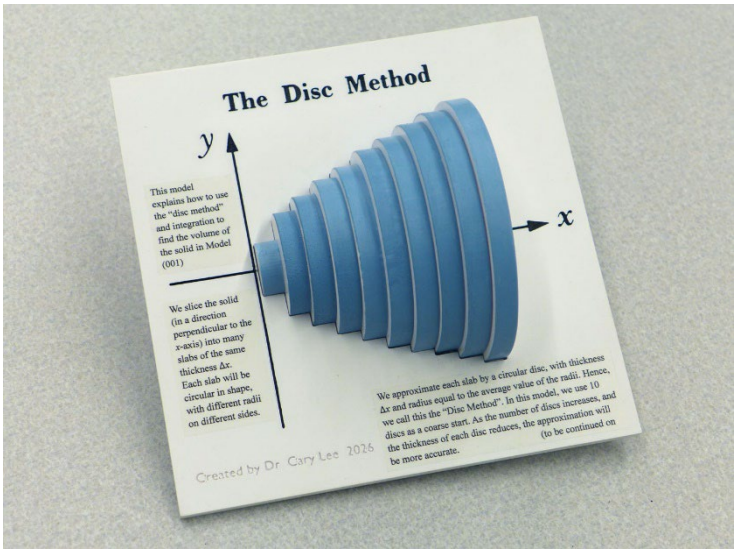

This is a typical solid generated by rotating a 2D region on the *xy*-plane about the *y*-axis. One quarter of it can be removed to show cross-section, and it will attach to the main solid by strong magnets.




The 10 shells are all separable, and yet they fit together nicely.

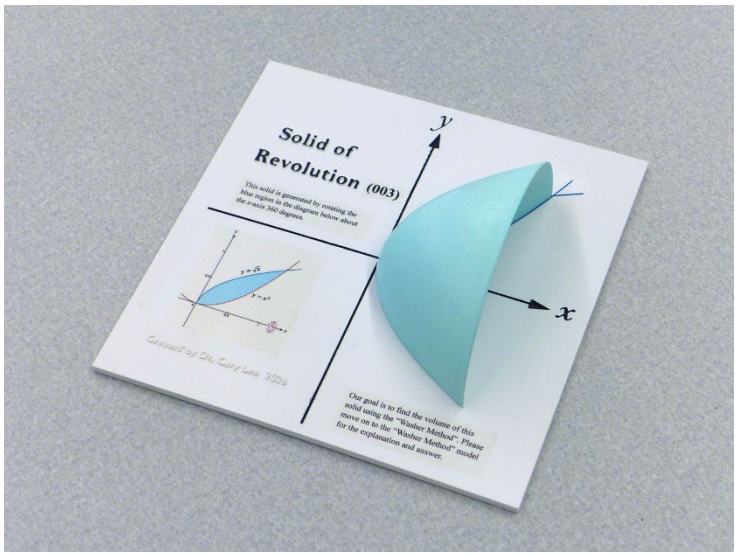
b) Solid of revolution and the disc method.

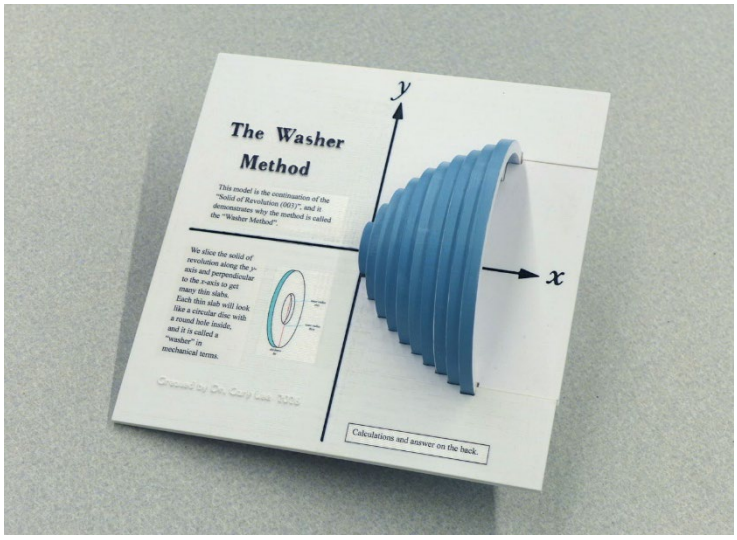



The 10 discs are all separable, and yet they fit together nicely.

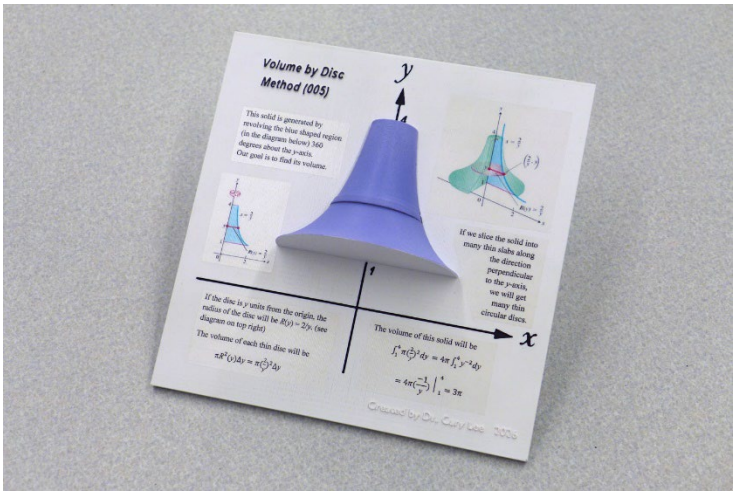
c) Solid of revolution and the washer method.




The 10 washers are all separable, and yet they fit together nicely.

d) A smaller model showing the disc method for another solid of revolution.

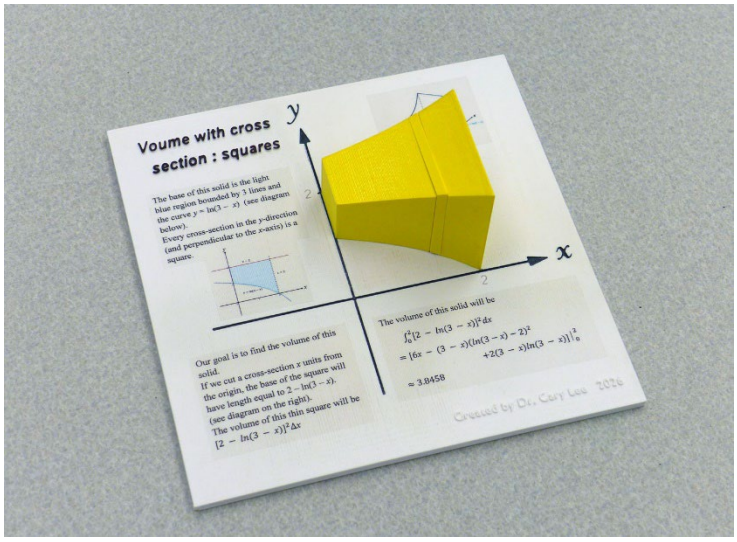



A circular disc can be removed to show the cross-section.

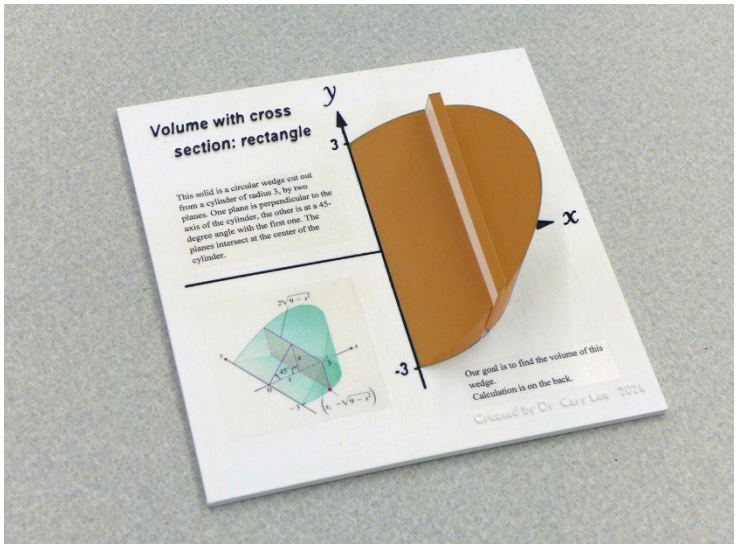
**Part II  Models for Solids with Special Cross-sections**

Each of these solids has a special base; and its cross-sections perpendicular to the *x*-axis take another special shape. The purpose of each model is to help students completely understand the shape of the solid and then find the volume. Each solid has one slice that can be removed to show the special cross-section. Calculations and answers are included in each model.

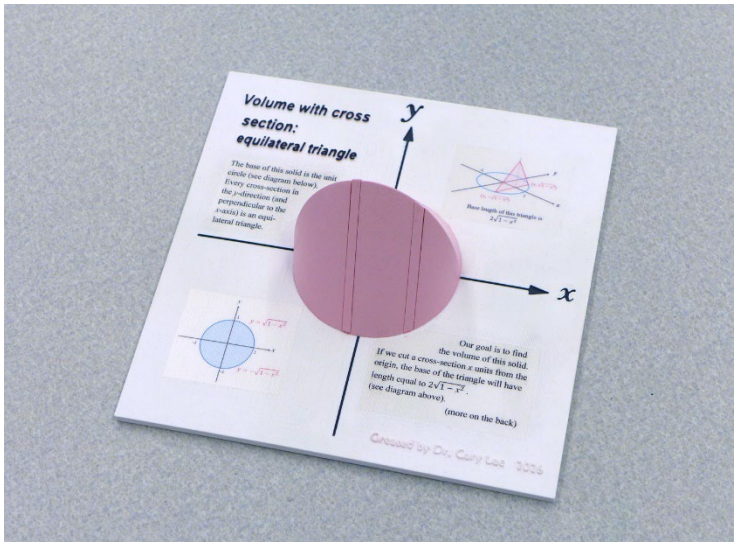
a) Solid with square cross-sections



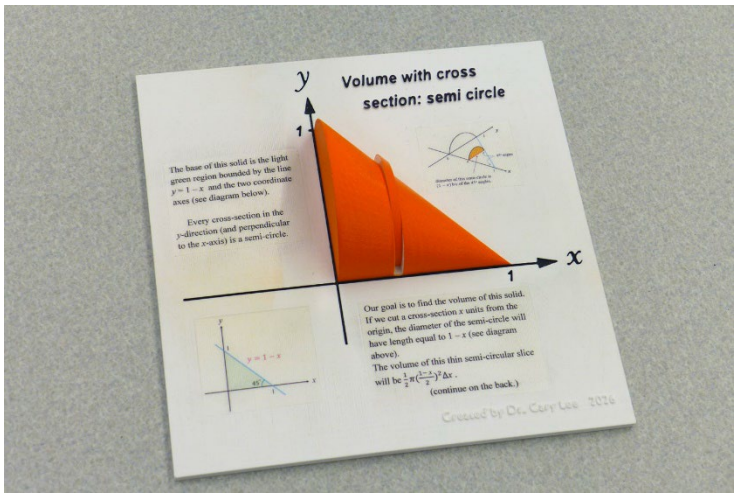

b) Solid with rectangular cross-sections



c) Solid with equi-triangular cross-sections

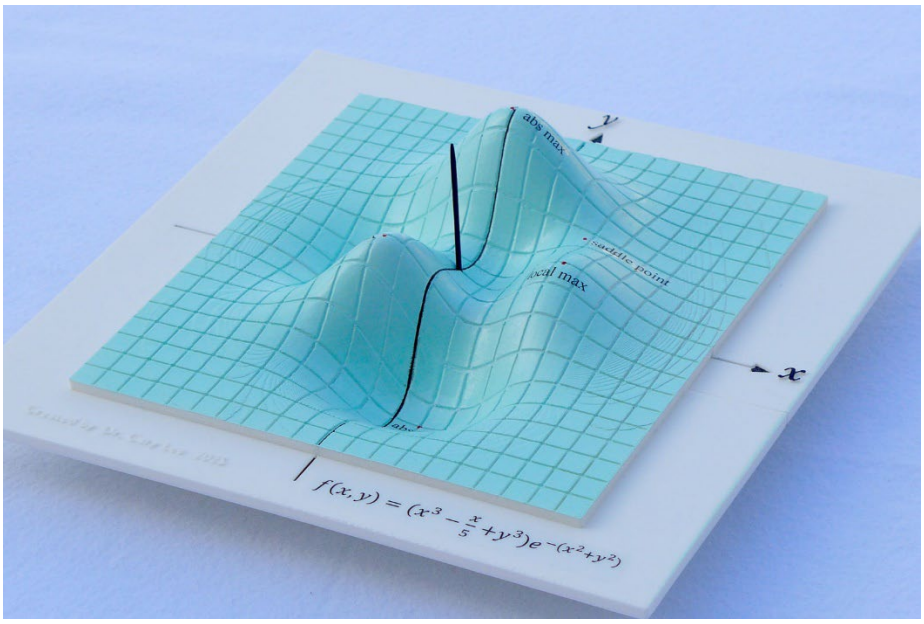



d) Solid with semi-circular cross-sections

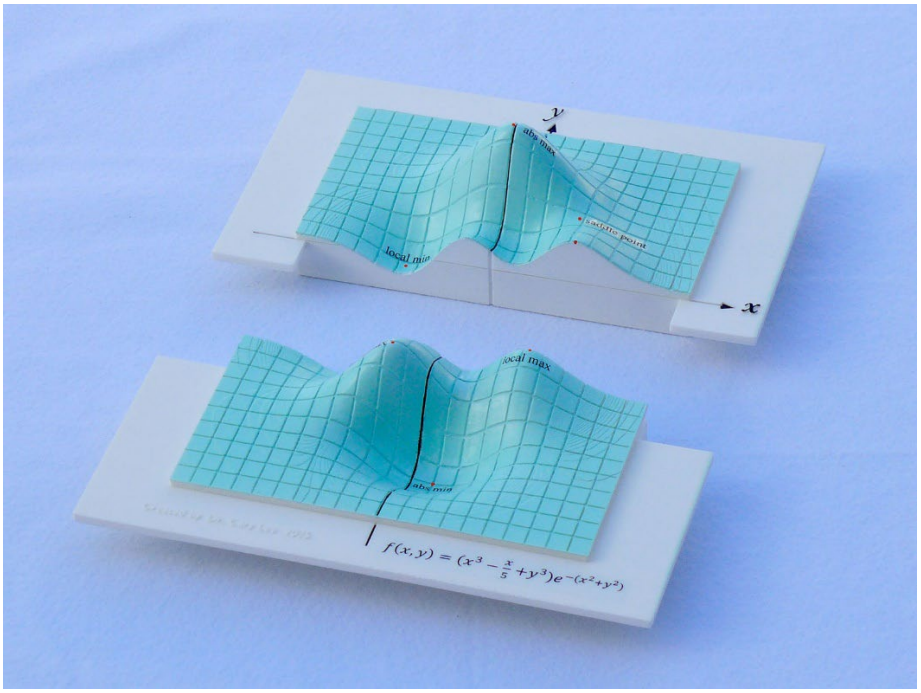


**Part III Models for functions of 2 variables.**

a) Graph of a generic smooth function of 2 variables as a smooth surface in 3D.

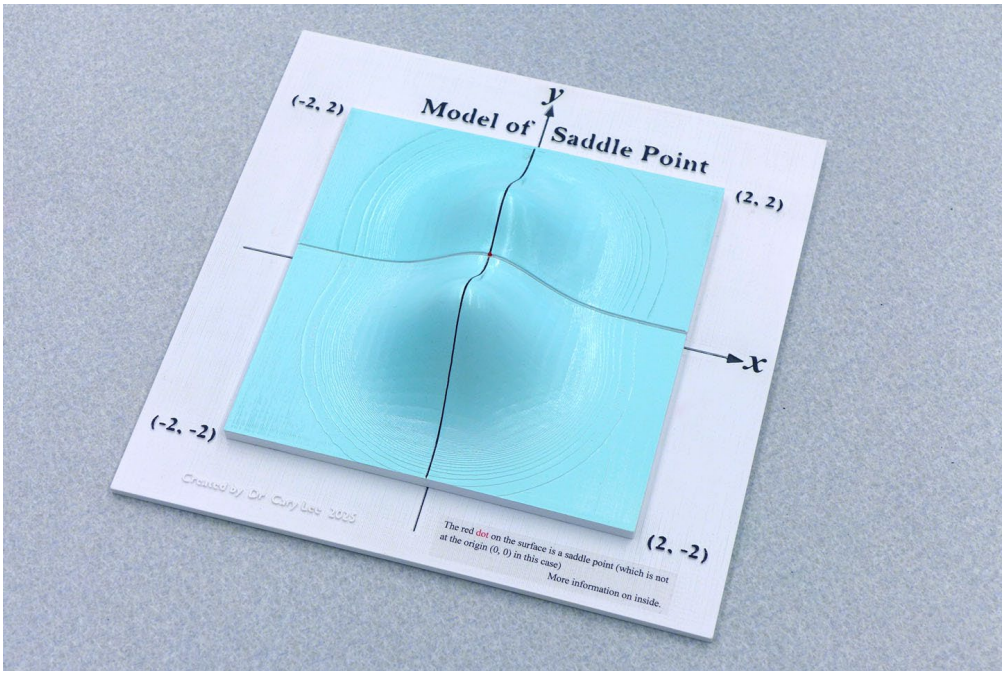



This model shows local and global extremums, as well as saddle points.

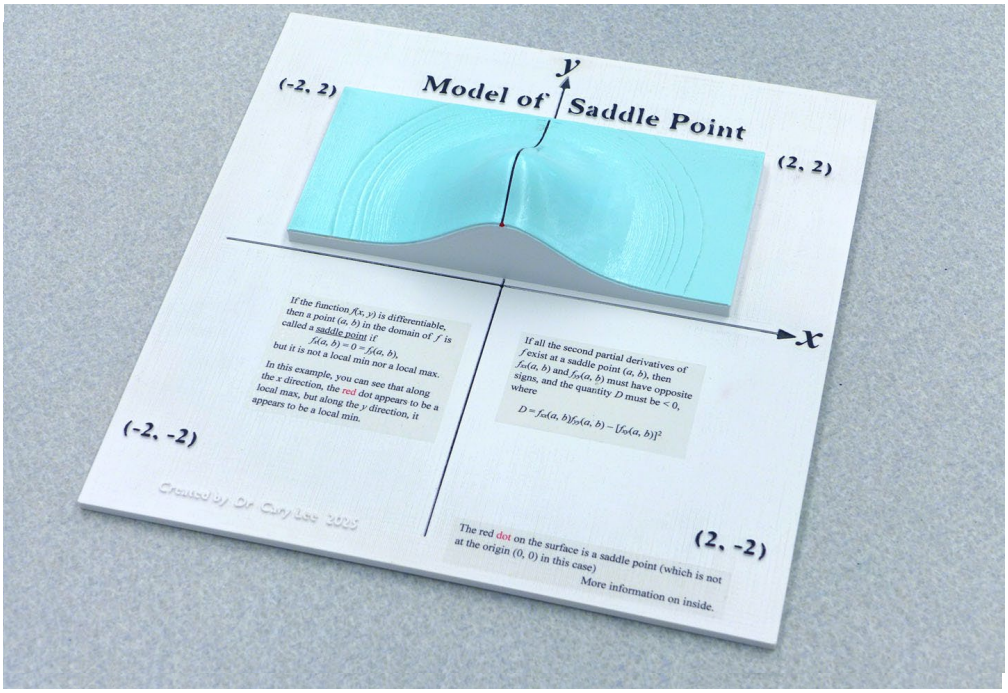



It can be pulled apart to show a cross-section along the *x*-axis. It can also be used to show partial derivatives, as well as directional derivatives.

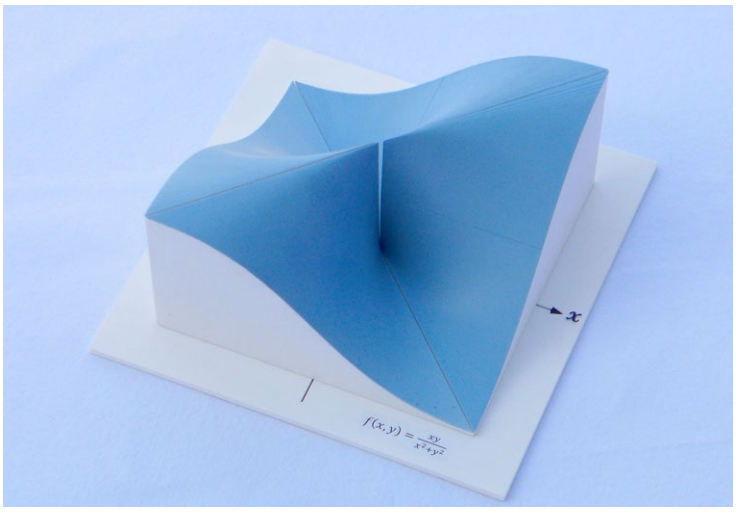
b) Saddle point – this model is specifically made to show case the features of a saddle point. But it can also be used to show that in functions of 2 variables, it is possible to have a differentiable function to have 2 local maximums but no local minimum. Such a situation will not be possible for a differentiable function of 1 variable.



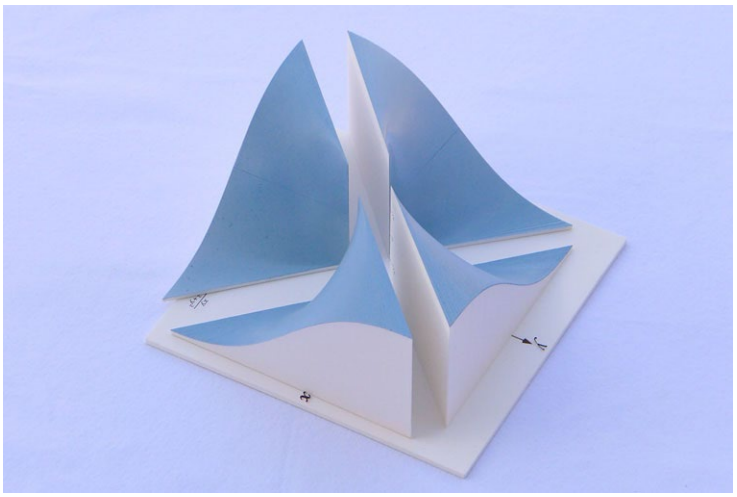

It can also be pulled apart to show a cross-section and explanations.

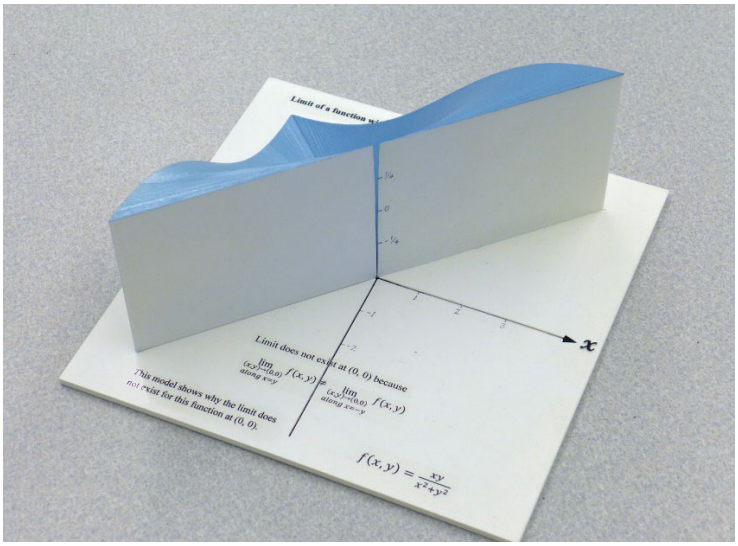


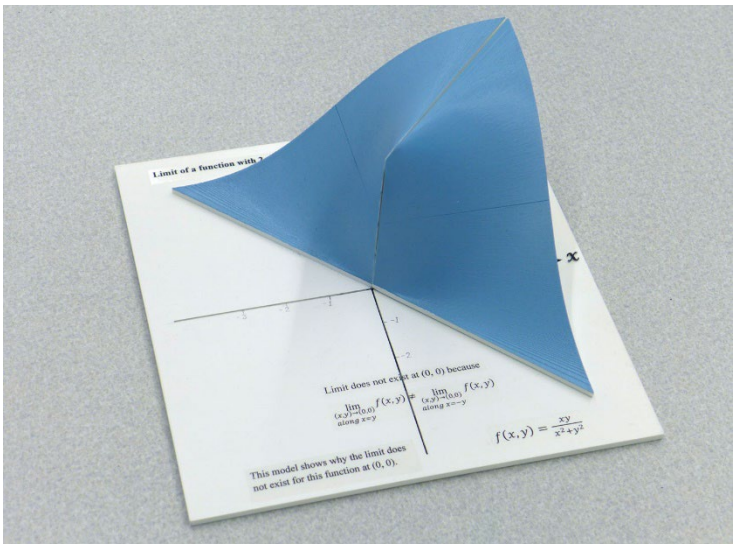
c) A function whose limit does not exist at (0, 0) even though it is differentiable everywhere else.



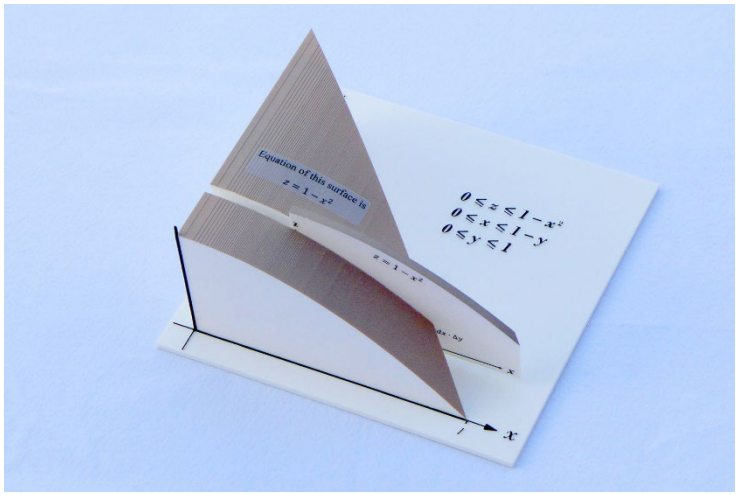

All 4 pieces can be separated; they are just held together by magnets.





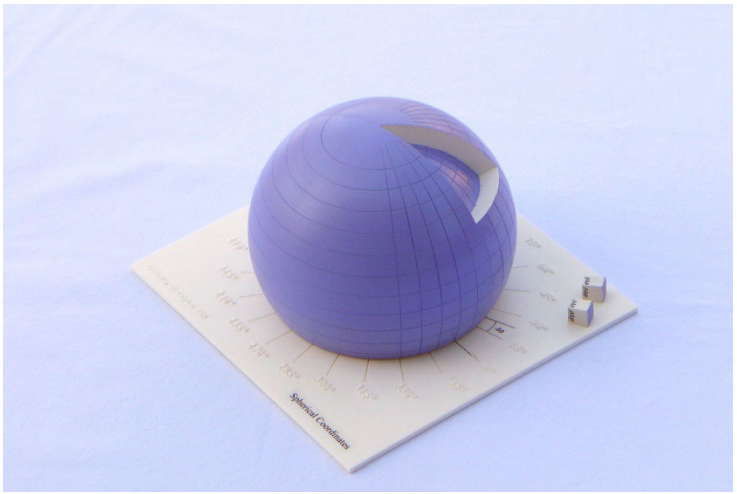



d) Triple Iterated Integral.
This model shows what it means to integrate along the *z*-direction first, and then the *x*-direction, and finally the *y*-direction. An application is to compute the mass of a solid when the density function is given.




A slice parallel to the *xz*-plane can be removed to show its shape.
We are in the process of making another one that slices parallel to the yz-plane.

e) Spherical coordinates




This model shows the positions of angle $\theta$ and angle $\varphi$, as well as the shape of each spherical wedge. These wedges are attached to the sphere by magnets.

